

# Effects of level inversion on the Coulomb displacement energies of the Ar-K isobaric analog state pairs

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## Abstract

The Coulomb displacement energies of the neutron-rich Ar-K isobaric analog state pairs with mass number  $A = 35 - 47$  are calculated within the relativistic mean field model and the effects of level inversion on the Coulomb displacement energies of the Ar-K isobaric analog state pairs are studied. The calculations are carried out in two cases, with and without consideration of the possible  $2s_{1/2}$  and  $1d_{3/2}$  proton level inversion of the neutron-rich Ar isotopes. Results show that the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of the neutron-rich Ar isotopes may reduce the Coulomb displacement energy by 0.06~0.17 MeV for the Ar-K isobaric analog state pairs. The results may provide a reference for experimental investigations of nuclear level inversion and a new test of the relativistic mean field model.

**Keywords:** Coulomb displacement energy; level inversion; relativistic mean field model; Ar isotopes.

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One of the long debated problem in nuclear physics is the existence of the level inversion and the relevant nuclear phenomena in exotic nuclei [1, 2]. The problem of nuclear level inversion has been studied with a variety of nuclear models and much progress has been achieved [3–12]. However, there are still many problems remaining unresolved; for instance, what are the real causes for the nuclear level inversion? In a recently published paper [13], we studied the possible proton  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of the neutron-rich Ar isotopes within the relativistic mean field model [14–21] and investigated the possibility to probe the level inversion using elastic electron-nucleus scattering. In addition to electron scattering, nuclear level inversion may also lead to other physical effects. It may, for instance, lead to the change of the Coulomb displacement energy [22–24] since the Coulomb displacement energies between isobaric analog states are due to isospin non-conserving forces, such as the Coulomb interaction between protons. Therefore, we would like to seek another possible way to detect the nuclear level inversion by investigating the effects of proton level inversion on the Coulomb displacement energies of the Ar-K isobaric analog state pairs.

In the present paper, we report our calculations of Coulomb displacement energy shifts brought about by the  $2s_{1/2}$  and  $1d_{3/2}$  proton level inversion of the neutron-rich Ar isotopes. Suggested by professor W. Mittig, we performed calculations on the Coulomb displacement energies of some neutron-rich Ar-K isobaric analog state pairs with  $A = 35 - 47$  within the relativistic mean field model with the NL-SH [20] and TM1/TM2 [21] parameters. The calculations were carried out in two cases: with and without consideration of the possible  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of the neutron-rich Ar isotopes. The results show that the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion may lead to a noticeable reduction of the Coulomb displacement energy, and this may provide a reference for experimental investigations of nuclear level inversion. In addition, the results may also provide a new test of the relativistic mean field theory.

## I. DESCRIPTION OF THE METHOD

Based on the formula

$$\Delta E_C = \frac{e}{2T} \int \rho_{exc}(\mathbf{r}) V_{core}(\mathbf{r}) d^3\mathbf{r} \quad (1)$$

given in Ref. [24], where  $V_{core}(\mathbf{r})$  is the Coulomb potential produced by the core protons and  $\rho_{exc}(\mathbf{r})$  is the distribution of the  $N - Z$  excess neutrons, the Coulomb displacement energies between the isobaric analog states and the respective nuclear ground states can be evaluated. We first carried out calculations on some arbitrarily chosen isobaric state pairs with mass number ranging from  $A = 27$  to  $A = 65$  to test the formula and the codes, and then performed calculations on neutron-rich Ar-K isobaric state pairs with mass number ranging from  $A = 35$  to  $A = 47$ .

In the calculation, the Coulomb potential field  $V_{core}(\mathbf{r})$  produced by the core protons and the distribution  $\rho_{exc}(\mathbf{r})$  of the  $N - Z$  excess neutrons are evaluated using one of the paired nuclei with the smaller proton number  $Z_<$ . For instance, for the Ar-K pair with  $A = 46$ , the Coulomb potential field  $V_{core}(\mathbf{r})$  is generated by the 18 core protons of Ar, that is, the 16 protons in  $1s_{1/2}$ ,  $2p_{1/2}$ ,  $2p_{3/2}$ ,  $1d_{5/2}$ ,  $2s_{1/2}$  orbitals and the 2 protons in the  $1d_{3/2}$  orbital; the excess neutron distribution  $\rho_{exc}(\mathbf{r})$  is given by the  $N - Z = 10$  neutrons with 2 of them in  $1d_{3/2}$  (or  $2s_{1/2}$ ) orbital and 8 in  $1f_{7/2}$  and higher orbitals. The Coulomb potential field  $V_{core}(\mathbf{r})$  and the excess neutron distribution  $\rho_{exc}(\mathbf{r})$  are both calculated using the single particle wave functions produced with the relativistic mean field model. For comparison and mutual verification, two sets of force parameters, the NL-SH [20] and TM1/TM2 parameters [21], are used, with TM1 used for the isobaric analog state pairs with  $A > 40$  and TM2 for those with  $A \leq 40$ .

## II. NUMERICAL RESULTS AND DISCUSSIONS

### A. The arbitrarily chosen isobaric analog state pairs with $A = 27 - 65$

To guarantee the validity of our calculation, we first test the method and the codes. We have arbitrarily chosen some isobaric analog state pairs for the test. In Table 1, we tabulate the calculated results and the experimental ones from Ref. [22], as well as the deviations along with some statistical results. The first and second columns of Table 1 are the mass numbers and corresponding specific isobaric analog state pairs that are chosen and calculated. The mass number ranges from  $A = 27$  to  $A = 65$ . The numbers in the brackets of the second column are the isospin values [22]. The symbol  $\Delta E_C$  in the third column denotes the experimental Coulomb displacement energies [22]; the symbols  $\Delta E_C^1$  in

the fourth column and  $\Delta E_C^2$  in the fifth column denote the calculated results of the Coulomb

TABLE I: The first column is the mass numbers of the isobaric pairs. The third column is the experimental Coulomb displacement energies [22], and the fourth and fifth columns are the calculated results with the NL-SH and TM1/TM2 parameters ( with TM1 parameters for the pairs with  $A > 40$  and TM2 parameters for the pairs with  $A \leq 40$  ). Columns six to nine are the deviations between the calculated results and the experimental ones. The last column is the renewed experimental data. The last three rows are the mean, minimum and maximum deviations.

A	isobaric pair $Z_{<}Z_{>}(T)$	$\Delta E_C$ Expt.	$\Delta E_C^1$ NL-SH	$\Delta E_C^2$ TM1(2)	$ \Delta E_C^1 - \Delta E_C $	$\frac{ \Delta E_C^1 - \Delta E_C }{\Delta E_C} \%$	$ \Delta E_C^2 - \Delta E_C $	$\frac{ \Delta E_C^2 - \Delta E_C }{\Delta E_C} \%$	$\Delta E_C^{\text{new}}$
27	Al-Si(1/2)	5.595	5.63751	5.52230	0.04251	0.75979	0.0727	1.29937	5.5947
29	Si-P(1/2)	5.725	5.69597	5.51713	0.02903	0.50707	0.20787	3.63092	5.72498
31	P-S(1/2)	6.178	6.05676	5.87302	0.12124	1.96245	0.30498	4.93655	6.18038
34	S-Cl(1)	6.274	6.42437	6.26309	0.15037	2.39672	0.01091	0.17389	6.27395
35	Cl-Ar(1/2)	6.748	6.76497	6.59962	0.01697	0.25148	0.14838	2.19887	6.74845
37	Ar-K(1/2)	6.931	7.09705	6.92081	0.16605	2.39576	0.01019	0.14702	6.9298
41	Ca-Sc(1/2)	7.278	7.20970	7.17352	0.0683	0.93844	0.10448	1.43556	7.27783
42	Ca-Sc(1)	7.208	7.19530	7.15578	0.0127	0.17619	0.05222	0.72447	7.20846
43	Sc-Ti(1/2)	7.650	7.53283	7.48842	0.11717	1.53163	0.16158	2.11216	7.64927
45	Ti-V(1/2)	7.915	7.85444	7.80318	0.06056	0.76513	0.11182	1.41276	7.91325
46	Ti-V(1)	7.834	7.84021	7.78598	0.00621	0.07927	0.04802	0.61297	7.83477
47	V-Cr(1/2)	8.234	8.17386	8.11559	0.06014	0.73039	0.11841	1.43806	8.22721
50	Mn-Fe(1)	8.934	8.82008	8.75047	0.11392	1.27513	0.18353	2.05429	8.91945
51	Mn-Fe(1/2)	8.802	8.80570	8.73400	0.0037	0.04204	0.068	0.77255	8.82376
53	Fe-Co(1/2)	9.085	9.10302	9.02239	0.01802	0.19835	0.06261	0.68916	9.07042
54	Co-Ni(1)	9.578	9.44369	9.36090	0.13431	1.40228	0.2171	2.26665	9.56907
55	Co-Ni(1/2)	9.476	9.40562	9.34438	0.07038	0.74272	0.13162	1.38898	9.47639
56	Mn-Fe(3)	8.596	8.55811	8.59486	0.03789	0.44079	0.00114	0.01326	8.59577
59	Cu-Zn(1/2)	9.874	9.73325	9.73302	0.14075	1.42546	0.14098	1.42779	9.92516
65	Ni-Cu(9/2)	9.221	9.21356	9.19391	0.00744	0.08069	0.02709	0.29379	9.21991
mean					0.06888	0.91	0.10918	1.46	
min					0.0037	0.043	0.00114	0.014	
max					0.16605	2.40	0.30498	4.94	

displacement energies with the NL-SH and TM1/TM2 parameters, respectively. Columns six and seven are the absolute and relative deviations of the calculated values with NL-SH parameters with respect to the experimental values. Columns eight and nine are the same as columns six and seven but for the results calculated with TM1/TM2 parameters. In Table 1, the experimental data for the  $^{56}\text{Mn}$ - $^{56}\text{Fe}$  and  $^{65}\text{Ni}$ - $^{65}\text{Cu}$  pairs were obtained with both  $^{56}\text{Fe}$  and  $^{65}\text{Cu}$  in the excited states [22]; for the other isobaric state pairs, the experimental data

were obtained with the paired nuclei both in the ground states [22]. The last three rows in Table 1 are the mean, minimum and maximum deviations, respectively. For intuitive comparison, the results are also shown in Fig. 1.

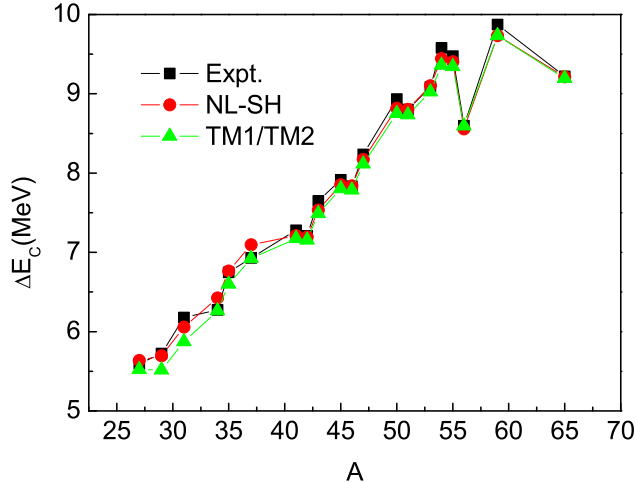


FIG. 1: Comparison of the calculated results with the experimental data [22]. The filled squares denote the experimental data; the filled dots, the calculated results with the NL-SH parameters; the filled triangles, the results with the TM1/TM2 parameters.

The experimental results of Ref. [22] were obtained based on a relatively old atomic nuclear mass data table given by Audi and Wapstra in 1995 [25]. To compare the present results with up-to-date experimental data, we renewed the experimental results based on the most recent atomic nuclear mass data given by Audi and Wapstra in 2012 [26]. The renewed experimental data are evaluated with the same formula [22],

$$\Delta E_C = M_{Z>} - M_{Z<} + \Delta nH, \quad (2)$$

as is used in Ref. [22], where  $M_{Z>}$  is the mass of the higher  $Z$  member nucleus of an analog pair and  $M_{Z<}$  is the mass of the lower  $Z$  member nucleus, and  $\Delta nH$  is the neutron-hydrogen mass difference of 0.782354 MeV. The renewed data are listed in the last column of Table 1 and the comparison with the calculated results is shown in Fig. 2.

Table 1, Fig. 1 and Fig. 2 show that the calculated Coulomb displacement energies are in good agreement with the experimental data. It is seen from Table 1 that the largest deviation of the results obtained with NL-SH parameters is less than 0.17 MeV and the mean

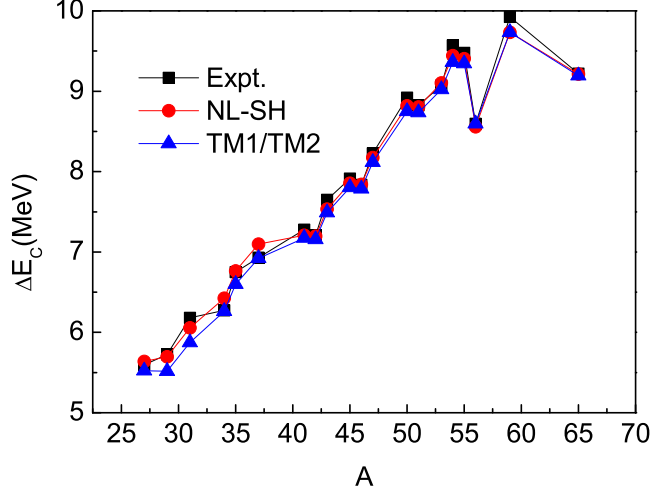


FIG. 2: Comparison of the calculated results with the renewed experimental data. The filled squares denote the renewed experimental data; the filled dots, the calculated results with the NL-SH parameters; the filled triangles, the results with the TM1(2) parameters.

deviation is less than 0.069 MeV; the corresponding relative deviations are 2.40% and 0.91%, respectively. For the results obtained with TM1/TM2 parameters, the largest absolute deviation is less than 0.31 MeV and the mean deviation is 0.11 MeV; the corresponding relative deviations are 4.94% and 1.46%. For the renewed data, it can also be easily obtained from Table 1 that the largest deviation of the results obtained with NL-SH parameters is less than 0.20 MeV and the mean deviation is less than 0.072 MeV; the corresponding relative deviations are 2.42% and 0.94%, respectively. For the results obtained with TM1/TM2 parameters, the largest absolute deviation is less than 0.31 MeV and the mean deviation is 0.12 MeV; the corresponding relative deviations are 4.98% and 1.47%. The experimental data are very well reproduced by the relativistic mean field model with the NL-SH and TM1/TM2 parameters. It is also found from Table 1 that the difference is very small (no more than 0.06 MeV) between the renewed data and the data of Ref. [22].

### B. The Ar-K isobaric analog state pairs

The Ar-K isobaric analog state pairs, which may be of some interests to the experimentalists, are calculated and discussed. The mass number of the chosen isobaric analog pairs ranges from  $A = 35$  to  $A = 47$ .

Calculations of the Ar isotopes within the relativistic mean field model show that there may exist the proton  $2s_{1/2}$  and  $1d_{3/2}$  level inversion for the Ar isotopes with  $A > 36$  [13]. Since the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion in Ar isotopes will change the distribution of the  $N - Z$  excess neutrons, we calculated the Coulomb displacement energies of the Ar-K isobaric analog state pairs in two cases. In one case (Case 1), the level inversion is neglected and two of the  $N - Z$  excess neutrons are assumed to occupy the  $1d_{3/2}$  orbital and the others occupy the  $1f_{7/2}$  and higher level orbitals; in another case (Case 2), the level inversion is taken into account and two of the  $N - Z$  excess neutrons are assumed to be in the  $2s_{1/2}$  orbital and the others in the  $1f_{7/2}$  and higher level orbitals. As has been shown that the renewed data only show very slight difference from the experimental results given in Ref. [22], so we will not give the renewed data in the following calculations for the Ar-K isobaric analog state pairs.

### 1. Case 1

In this case, the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion for the Ar isotopes with  $A > 36$  is not considered, i.e., the distribution  $\rho_{exc}(\mathbf{r})$  of the  $N - Z$  excess neutrons are evaluated with two of the  $N - Z$  excess neutrons in the  $1d_{3/2}$  orbital and the others in the  $1f_{7/2}$  and higher level orbitals. The results are listed in Table 2.

In Table 2, the experimental data are from Ref. [22]; the experimental result of  $^{37}\text{Ar}$ - $^{37}\text{K}$  was obtained with  $^{37}\text{Ar}$  and  $^{37}\text{K}$  both in the ground states; the others were obtained with either Ar or K nucleus in the excited states, or both of them in the excited states. The last row in Table 2 is the results of the  $^{47}\text{Ar}$ - $^{47}\text{K}$  pair, whose experimental data are unavailable. The results are also plotted in Fig. 3, with panel (a) plotted on the same scale as Fig. 1 for comparison and panel (b) on a smaller scale to show some details for discussion and comparison. The plots show that the general trends of variation of the calculated results with the two sets of parameters are very similar to that of the experimental data.

However, on a smaller scale it is seen from panel (b) that in this case the TM1/TM2 parameters produce better results for Ar-K pairs than the NL-SH parameters, especially for those near the stability line. The results obtained with the NL-SH parameters are generally a little larger than the experimental values.

TABLE II: The experimental Coulomb displacement energies of the Ar-K isobaric pairs and the calculated results in the case that the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of Ar isotopes is not considered.

A	isobaric pair $Z_{<} - Z_{>}(T)$	$\Delta E_C$ Expt.	$\Delta E_C^1$ NL-SH	$\Delta E_C^2$ TM1(2)
35	Ar-K(3/2)	7.091	7.14234	6.96404
36	Ar-K(1)	6.977	7.12519	6.96379
37	Ar-K(1/2)	6.931	7.09885	6.92676
38	Ar-K(1)	6.826	7.051	6.87161
39	Ar-K(3/2)	6.764	6.94675	6.77747
40	Ar-K(2)	6.671	6.87715	6.71587
41	Ar-K(5/2)	6.64	6.82366	6.66754
45	Ar-K(9/2)	6.515	6.66499	6.59396
46	Ar-K(5)	6.553	6.61714	6.54256
47	Ar-K	—	6.53535	6.47139

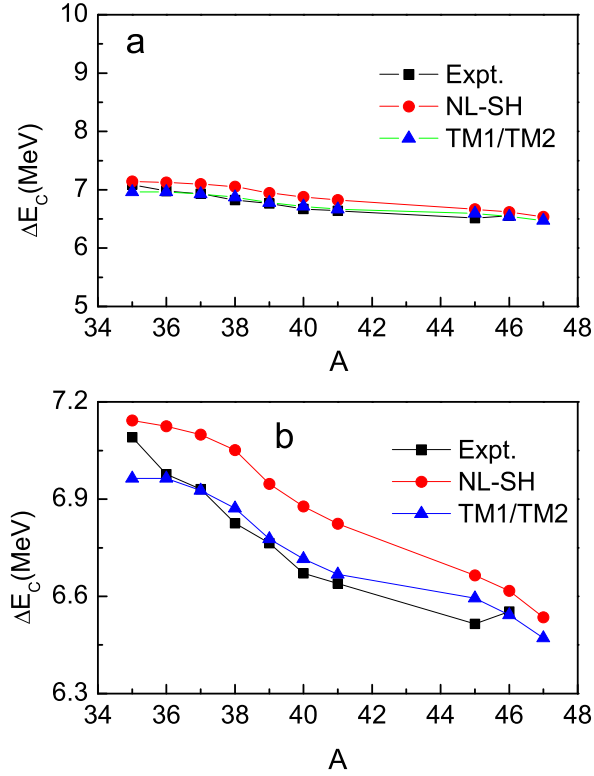


FIG. 3: The experimental data and calculated results for the Ar-K isobaric analog state pairs without consideration of the possible  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion of Ar isotopes. In panel (a) the results are plotted on the same scale as Fig. 1 and in panel (b) the results are plotted on a smaller scale.



## 2. Case 2

In this case, the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion for the Ar isotopes with  $A > 36$  is taken into account in the calculation of the Coulomb displacement energies. The distribution  $\rho_{exc}(\mathbf{r})$  of the  $N - Z$  excess neutrons are evaluated with two of the  $N - Z$  excess neutrons in the  $2s_{1/2}$  orbital and the others in the  $1f_{7/2}$  and higher level orbitals. The numerical results are given in Table 3 and plotted in Fig. 4 on the same scales as Fig. 3.

TABLE III: The experimental results of the Ar-K isobaric pairs and the calculated results in the case that the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of Ar isotopes is considered.

A	isobaric pair	$\Delta E_C$	$\Delta E_C^1$	$\Delta E_C^2$
	$Z_{<} - Z_{>}(T)$	Expt.	NL-SH	TM1(2)
35	Ar-K(3/2)	7.091	7.14234	6.96404
36	Ar-K(1)	6.977	7.12519	6.96379
37	Ar-K(1/2)	6.931	7.03958	6.79593
38	Ar-K(1)	6.826	6.98583	6.75248
39	Ar-K(3/2)	6.764	6.83119	6.62398
40	Ar-K(2)	6.671	6.74552	6.55085
41	Ar-K(5/2)	6.64	6.68764	6.50013
45	Ar-K(9/2)	6.515	6.53733	6.47287
46	Ar-K(5)	6.553	6.49264	6.42513
47	Ar-K	—	6.41374	6.35766

It is seen from Fig. 4 that, in this case, the results are lowered or shifted down a little in comparison with the results of Case 1. The experimental results nearly fall between the theoretical results obtained from the two parameter sets; the deviations between the results obtained with the NL-SH parameters and the experimental ones are reduced, yet the results given by the NL-SH parameters are still a little larger than the experimental results; however, the deviations of the results given by the TM1/TM2 parameters from the experimental ones are enlarged slightly.

To show the effects of the  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion on the Coulomb displacement energy, we calculated the differences of the results between Case 1 and Case 2 for either parameter set. The results are shown in Table 4 and plotted in Fig. 5, where

$$\delta = \Delta E_C^1(case1) - \Delta E_C^1(case2) \quad (3)$$

for the NL-SH parameters, and

$$\delta = \Delta E_C^2(case1) - \Delta E_C^2(case2) \quad (4)$$

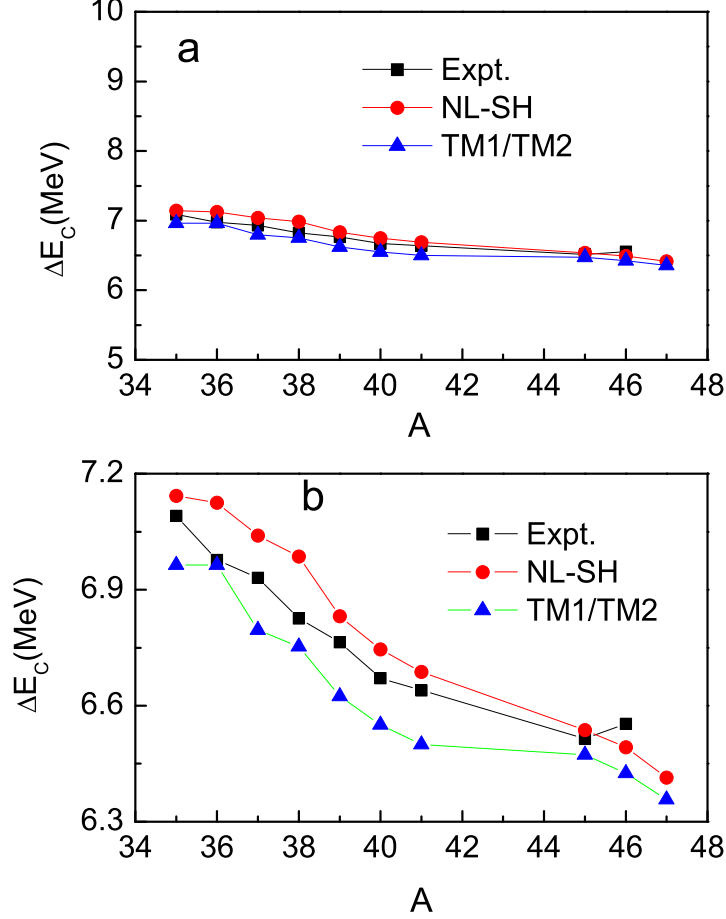


FIG. 4: The same as Fig.3 but for the results obtained considering the possible  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion of the Ar isotopes.

for the TM1/TM2 parameters.

The results in Table 4 show that the  $2s_{1/2}$  and  $1d_{3/2}$  level inversion of Ar isotopes leads a reduction to the Coulomb displacement energies by 0.06~0.17 MeV for the Ar-K isobaric pairs, and the largest reduction, which occurs to the Ar-K pairs near  $A = 41$ , is about 0.17 MeV. A circumstantial evidence for our result was given in Ref. [23], where the effects of the halo on the Coulomb displacement energy for the isobaric analog state of  $^{11}\text{Li}$  were investigated, and the halo is found to reduce the Coulomb displacement energy by 0.100~0.200 MeV. The plots in Fig. 5 show that the trends of variation of the differences  $\delta$  given by the two sets of parameters are approximately the same.

Finally, a few remarks on the result of the  $^{46}\text{Ar}$ - $^{46}\text{K}$  pair should be made. It can be found from panel (b) of Fig. 3 or Fig. 4 that the experimental result of the  $^{46}\text{Ar}$ - $^{46}\text{K}$  pair seems to

TABLE IV: The differences of the calculated Coulomb displacement energies between the two cases for the two sets of parameters.

A	isobaric pair	$\delta$	$\delta$
	$Z_{<-Z_{>}(T)$	NL-SH	TM1(2)
35	Ar-K(3/2)	0	0
36	Ar-K(1)	0	0
37	Ar-K(1/2)	0.05927	0.13083
38	Ar-K(1)	0.06517	0.11913
39	Ar-K(3/2)	0.11556	0.15349
40	Ar-K(2)	0.13163	0.16502
41	Ar-K(5/2)	0.13602	0.16741
45	Ar-K(9/2)	0.12766	0.12109
46	Ar-K(5)	0.1245	0.11743
47	Ar-K	0.12161	0.11373

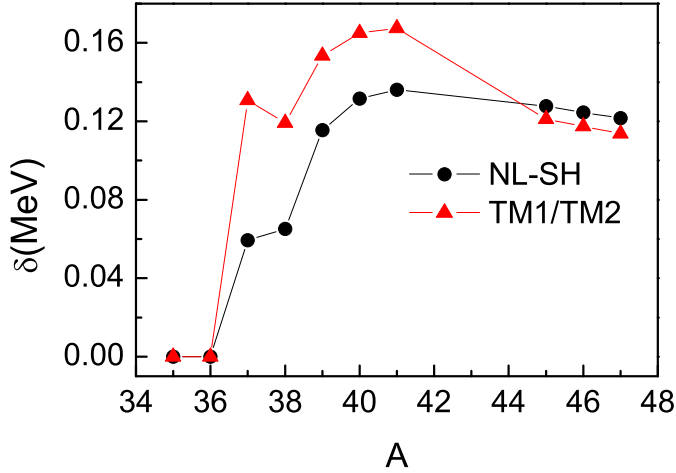


FIG. 5: Differences of the calculated results of the Ar-K isobaric analog state pairs between Case 1 and Case 2 for the two parameter sets.

have a “strange behavior”; it shows a deviation, an up warp, from the gradually decreasing trend of variation of the Coulomb displacement energies of the Ar-K pairs with the increase of mass number. This deviation is not reproduced by either set of parameters in either cases. This may imply either that the description of very neutron-rich Ar-K pair by the relativistic mean field model is not so good as that of the nuclei near the stability line or that there is a lack of accuracy in the experimental result for the  $^{46}\text{Ar}$ - $^{46}\text{K}$  pair, since the excitation energy for  $^{46}\text{K}$  given in Ref. [22] is just an approximate value ( $\approx 11.470$  MeV).

### III. SUMMARY

Within the relativistic mean field model, the Coulomb displacement energies of the Ar-K isobaric analog state pairs with mass number  $A$  ranging from 35 to 47 are calculated directly with the Coulomb potential generated by the core protons and the distribution of the  $N - Z$  excess neutrons within the relativistic mean field model. The calculation is carried out in two cases: In one case (Case 1), the  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion is not considered, i.e., two of the  $N - Z$  excess neutrons are assumed to be in the  $1d_{3/2}$  orbital; in another case (Case 2), the  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion is taken into account, i.e., two of the  $N - Z$  excess neutrons are assumed to be in the  $2s_{1/2}$  orbital. The results show that the  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion may lead a reduction of no more than 0.17 MeV to the Coulomb displacement energy, and the largest reduction may occur to the  $^{40-42}\text{Ar}$ - $^{40-42}\text{K}$  pair. The results may provide a reference for experimentally studying the  $2s_{1/2}$  and  $1d_{3/2}$  state level inversion. In addition, the  $^{46}\text{Ar}$ - $^{46}\text{K}$  pair is also discussed for its “seemingly strange” experimental value.

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